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On the Mechanical Stretching of Liquids: An Experimental Determination of the Volume-Extensibility of Ethyl-Alcohol

A. M. Worthington

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VIII. *On the Mechanical Stretching of Liquids: an Experimental Determination of the Volume-Extensibility of Ethyl-Alcohol.*

By A. M. Worthington, *M.A., Professor of Physics and Head Master of the Royal Naval Engineering School, Devonport.*

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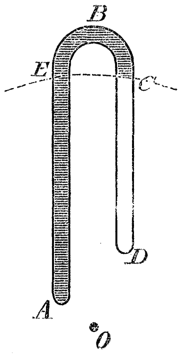
[Plate 10.]

Three methods are known by which a liquid may be subjected to a bodily tension.

(1) *The method of the inverted barometer*, familiar to most physicists, by which, with care, a mercury column of many times the barometric height may be supported by its adhesion to the top of the tube. In such a column the hydrostatic pressure is negative above the barometric height, or the liquid above this level is in a state of tension. This tension increases with the height and is propagated in all directions to the walls of the tube. When the upper part of the tube is made elliptical in cross-section and of thin glass, its yielding to the inward pull may be easily observed.

(2) *The centrifugal method*, devised by Professor Osborne Reynolds, in which a U-tube, ABCD, of glass, closed at both ends, contains air-free liquid, ABC, and vapour, CD. This tube is fixed to a suitable board and whirled about an axis, O, a little beyond the end, A, and perpendicular to the plane of the board. If CE (see figure) be the arc of a circle described about O, then while rotation continues the liquid between E and A is in a state of tension, increasing from zero (if we ignore the vapour-pressure) at E to a maximum at A. By this method Professor Osborne Reynolds has subjected water to a tension of about 5 atmospheres or 72·5 pounds per square inch, while the author, experimenting in the Cavendish Laboratory in



1886, succeeded in reaching, with alcohol, a tension of 7·9 atmospheres or 116 pounds per square inch, and, with strong sulphuric acid, 11·8 atmospheres or 173 pounds per square inch.

(3) *The method of cooling*, discovered by M. BERTHELOT, and described by him in a paper entitled "Sur la Dilatation Forcée des Liquides," published in 1850 ('Ann. de Chimie,' vol. 30, 1850, pp. 232–237), by which he succeeded in obtaining a great variety of liquids in a state of very considerable mechanical extension, the amount of which he appears, however, rather to have estimated than to have measured, but which, according to his estimate, was as much as $\frac{1}{120}$ of the whole volume in the case of water, $\frac{1}{93}$ in the case of alcohol, and $\frac{1}{59}$ in that of ether.

In M. BERTHELOT'S experiments the liquid, freed of air by long boiling, nearly filled a straight, thick-walled glass tube, the small residual space being occupied by its vapour. When slightly heated the liquid expanded and filled the whole tube, but on being again cooled remained extended, still filling the tube, of which it at last let go its hold with a loud metallic click, when the bubble of vapour re-appeared. It was from the length of this bubble that the extension was calculated.

It will be observed that methods (1) and (2) afford measures of the tensile stress, but not of the strain or extension; while, on the other hand, the method (3) affords a measure of the strain but not of the stress. The object of the present paper is to describe the process by which, after a great variety of trials made during the past six years, I have succeeded in what, so far as I am aware, has not been previously attempted, viz., in obtaining simultaneous measures of stress and strain in a liquid under tension. The measures are not, indeed, as numerous as could be wished, for reasons that will appear in the sequel, but they are fairly consistent, and mark a stage in an investigation with which I hope to proceed further, and to which I am anxious to attract the attention of more skilful experimenters.

That a liquid can pass into and exist stably in a state of tension without any breach whatever of physical continuity has been denied or questioned by eminent physicists,* and the contrary is commonly asserted by writers on Hydrodynamics.† The experiments to be described will be found, I think, to remove the last possibility of doubt upon the matter.

Method of Experimenting.

The mode of subjecting the liquid to tensile stress is essentially that of BERTHELOT, and may be briefly described as follows:—The liquid is contained in a strong closed glass vessel which it nearly fills at the ordinary temperature of the air. The small space not filled by liquid is occupied only by its vapour. Dissolved air, and especially the film of air which at first lies between the liquid and the walls of the vessel, is got rid of, as far as possible, by prolonged boiling before the vessel is sealed up.

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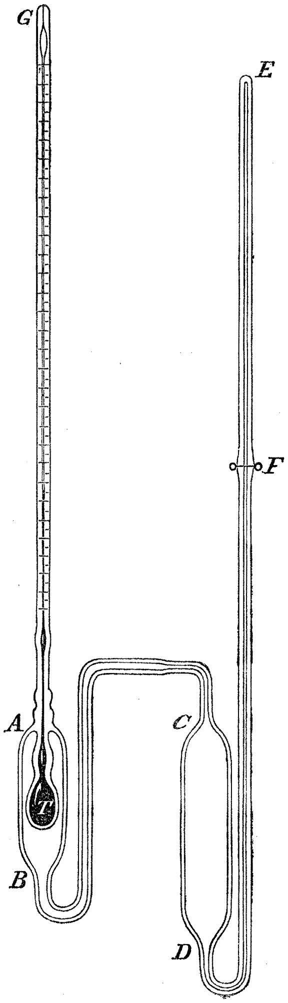
* See BALFOUR STEWART, 'Elementary Physics,' § 69, p. 70.

† See LAMB, 'Treatise on the Motion of Fluids,' § 9, p. 7.

The temperature of part of the vessel and its contents is now raised. This causes the liquid to expand and fill the whole; if the rise of temperature continued the vessel would burst; but before the pressure becomes dangerously great the liquid is cooled again by means of ice-cold water. It now, however, adheres so firmly to the walls of the vessel that, though cooled, it cannot contract, and it remains extended or stretched, tugging at the walls until at last, as the cooling proceeds, the tension becomes so great that the liquid lets go its hold, and releases the glass walls with a loud metallic click, and itself springs back to the smaller volume appropriate to the temperature and to the pressure of its saturated vapour.

Fig. 1.



Whether the cohesion of the liquid for itself or its adhesion to the glass is first overcome is not decisively determined, but all indications point rather to the latter than to the former supposition.

The form and dimensions of the vessel used in my experiments are given by the accompanying diagram, which is drawn to the scale of one quarter.

The vessel consists of two very strong cylindrical bulbs, AB and CD, with rounded ends, connected by a tube of narrow bore bent in the manner shown.

The walls of the bulbs were about 2 millims. thick and the bore of the connecting tube was about 2·5 millims. internal diameter. The diagram being drawn to scale gives any other necessary information as to dimensions.

DE is a narrow tube of very uniform bore (2.2 mm.), across which, at F, was sealed, with as little disturbance of the bore of the tube as possible, a very fine platinum wire.

Measurement of the Tension.

In order to measure the tension of the liquid at any instant, an ellipsoidal bulb T filled with mercury, and provided with a narrow graduated capillary stem AE is sealed into the bulb AB. This bulb had been previously subjected to various measured pressures in an hydraulic press up to 60 atmospheres, and the corresponding rise of the mercury in the stem noted. This was found, within the limits of observational error, to be proportional to the pressure applied. This rise is due to the diminution in capacity of the bulb, which becomes less spherical under the external pressure.

On the other hand, when the surrounding liquid is in a state of *tension*, it tugs at the walls of this bulb and makes it more spherical and of greater capacity, and since the bulb is thick-walled and fairly rigid, and the alteration of volume only a very small fraction of the whole, it appears quite safe to assume that the enlargement produced by a given tension is equal to the diminution of volume that is produced by an equal pressure applied *over the same surface.*

This instrument for measuring the tension I call the Tonometer. In that actually used the relation between bulb and tube was such that a pressure of 1 atmosphere of 15 lbs. to the square inch, as measured by a Bourdon gauge, gave a rise of 3·296 millims. The actual observations from which this value was determined were the following:—

Date.	Pressure employed (Bourdon gauge).	Resulting rise in tonometer reading for 1 atmosphere of 15 pounds per sq. inch.
	atmospheres.	millims.
Nov. 29, 1889	8	3·25
" "	8	3·375
" "	8	3·3125
Dec. 13, 1889	8	3·125
" "	. .	3·187
" "	10	3·450
" "	10	3·333
April 29, 1891	12	3·333
	Mean .	3·296

The Bourdon gauge used was employed for other observations to be described later, and when all were completed it was re-tested for me by Mr. C. F. CASELLA, by means of a mercury column, at the beginning of December, 1891, up to 14 atmospheres, with the following results :—

Bourdon gauge.		Mercurial column.
atmospheres.	pounds per sq. inch.	pounds per sq. inch.
0	0	0
2	30	30
4	60	62
6	90	89
8	120	121·5
10	150	151·5
12	180	179
14	210	209

The deviations from the original calibration up to this range will be observed almost to fall within the errors of reading, and by an independent comparison, which I made myself in the same month, of this gauge with two official standard gauges, by E. BOURDON, belonging to the Admiralty, I could not detect with any certainty any appreciable divergence of the readings, which I think may therefore be taken as correct within 1 per cent.

It may be mentioned, that in May, 1889, pressures of 36 gauge-atmospheres had been applied to the tonometer, giving a mean rise of 3·56 millims. per atmosphere, but in December, 1891, the higher readings of the gauge were found to be as much as 8 per cent. too low. The application of this correction reduced the reading to 3·275 millims. which accords well with the results already quoted, but on account of the doubt as to whether the gauge was ever correct at this part of the scale, it seemed better to exclude these observations altogether in determining the tonometer calibration, and to rely only on those within the range that I had myself tested.

It was afterwards observed by using the tonometer as a thermometer that the rise of the mercury due to a rise of temperature of 1°·85 C. was 2 centims., showing that the deformation of the bulb due to the maximum tension afterwards obtained, viz. 17 atmospheres, amounted to only about 1/1000th of its whole volume.

Measurement of the Strain or Extension.

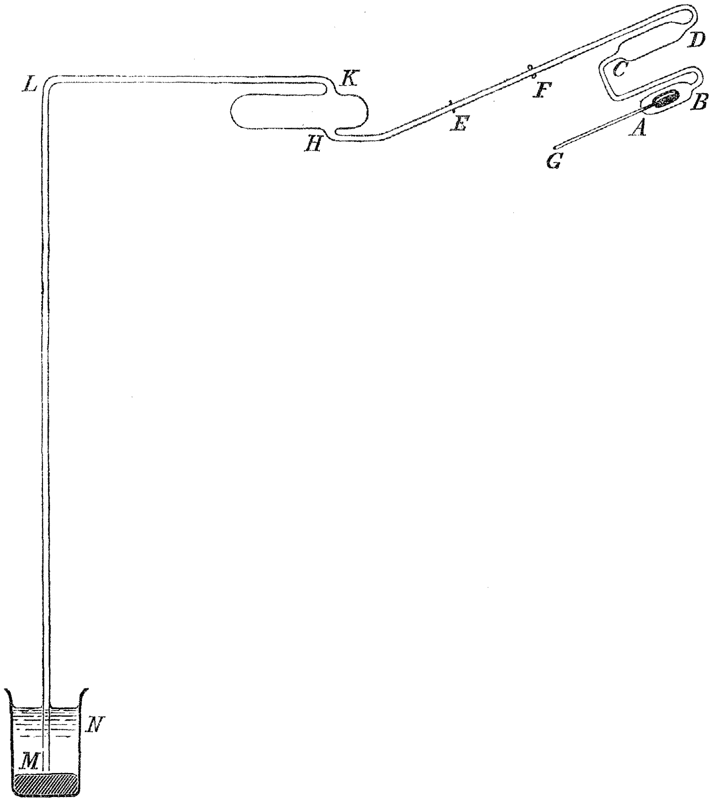
In order to ascertain the extension at any instant, the liquid was caused to let go its hold, and thus spring back to its unstretched volume, and then the volume of the space left empty of all but vapour was measured.

For this purpose the straight tube AB was traversed at F by the fine platinum wire, already mentioned, running across the centre of the tube. One of the projecting ends of this wire was connected by a stouter copper wire to one pole of an electric storage cell, and the other could at pleasure be touched by the bare end of a similar stout wire from the other pole. By this means the fine platinum wire could be heated suddenly to redness, when the liquid which was tugging at it immediately let go its hold. The bubble of vapour thus caused to appear had its upper end at the wire, and extended below to a distance along the fine tube, which could be accurately measured against a scale placed, to avoid parallax, on a mirror behind the tube.

The relation of the bore of the tube to the whole internal volume of the apparatus had been previously ascertained, and thus the fraction of the whole volume which the extension represents was known.

This extension is, however, only *apparent*, and it is necessary to determine and to subtract from it the amount by which the volume of the containing vessel has been diminished by the inward pull. The separate operation for determining this correction will be described later. I shall now explain the mode of filling and sealing the apparatus, and of conducting an experiment, with the precautions and minor corrections necessary for securing good measures of the extension.

Fig. 2.



Method of Filling the Apparatus

After thoroughly cleaning the apparatus with, first, a solution of potassium hydrate, then dilute hydrochloric acid, and then finally with distilled water, it is sealed at E to a second glass bulb HK, with arms HE, KLM, as shown in fig. 2.

The branch LM, which is more than the length of the mercury barometer, hangs vertically. For the sake of greater clearness of representation the diagram shows the bulbs HK, CD, and AB with their axes in the same plane, but in practice it is more convenient when making the connection at E to turn the apparatus that is to be filled through a right angle about EF as axis, since in this position liquid contained in either bulb can be more easily boiled by means of a small Bunsen flame.

The liquid to be experimented on is placed in a large beaker N over a layer of quite clean mercury 2 or 3 centims. deep, and into this beaker the open end M of the tube ZM dips, not quite reaching the mercury. After the liquid in N has been boiled to expel dissolved air, it is allowed to enter the bulbs HK, CD, and AB, and is then boiled in each simultaneously by suitable flames.* Thus, on the removal of the flame from beneath AB, air-free liquid from CD enters and fills it; and when that in CD ceases to boil, air-free liquid from GHK enters and fills CD, and so on.

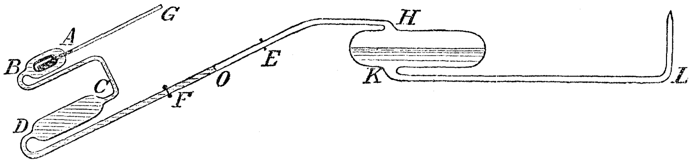
The process of alternately boiling out and filling is continued, with pauses, for several hours, till the residual bubbles disappear very quickly on cooling, and the tendency to boil by bumping threatens to endanger the apparatus.

Then while the liquid in HK is kept gently boiling, the tube LM is heated nearly to redness, just below L, in order to drive off attached air which is carried off in the stream of passing vapour.

Then when this tube has again cooled, and the vessel HK is about half-full, the vessel N is raised so as to submerge the open end M below the mercury, which, when all is cool, rises to nearly barometric height. Then after any liquid that has condensed above the mercury in ML has been, by judicious heating, driven over into the bulb GHK, the tube LM is heated just below L to the softening point, and allowed to close under atmospheric pressure. This process of sealing invariably liberates gas either from the surface of glass or by decomposition of the vapour of the liquid. It appears, however, to be less in quantity when the tube near the sealing point has been previously subjected to strong heating in the manner described.

The portion of the apparatus thus cut off from the barometer tube is now turned about a horizontal axis along GE, through 180°, into the position shown in fig. 3,

Fig. 3.



and by careful heating the liquid filling the tube EF is boiled over into the bulb HK till when all has cooled to the temperature of the room, the surface stands at some

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* It is important that the stem AG of the tonometer shall be provided with an enlargement at the end in order that the instrument may not burst when the mercury in the ellipsoidal bulb is raised to the temperature of the surrounding hot liquid.

suitable place O between E and F, 2 or 3 centims. from the extremity E of the uniform capillary tube. Then the liquid in the bulbs AB, CD is cooled by ice or cold water, so that the surface recedes towards, or even beyond F. Next the liquid in HK is cooled so as to diminish the vapour-pressure and cause a draught of vapour from the surface of the warmer liquid in the tube, and while this draught continues, the tube EF is heated just between O and E, and allowed to close under atmospheric pressure. Gases formed or liberated in this operation are for the most part carried over into the large bulb GHK by the draught of vapour.

The length of tube which it is desirable to leave empty above the surface of the liquid at O when all is at the temperature of the room again, should be just, but not more than, sufficient to secure that the instrument shall not be in danger of bursting when left to itself and exposed to unavoidable changes of temperature in the room where it is placed.

The trace of gas that is liberated in sealing is readily dissolved in the liquid, but on this account the liquid in the tube EF becomes unduly charged with gas, and does not, at first, adhere very well to the walls. It is well, therefore, by raising the temperature of the bulbs, to compress the residual vapour into a bubble small enough to be floated along the tube into the bulbs, where any residual gas it may contain is dispersed through a large mass of liquid.

The apparatus is now ready for experimenting. For this it is set in the erect position shown in fig. 1. A beaker of water of the temperature of the room is placed round the bulb containing the tonometer to secure it from temperature changes. A second beaker, containing warm water, is placed round the other bulb, while a third beaker, which can be quickly substituted for this, contains ice and ice-cold water. Plane mirrors are placed behind the two tubes to prevent parallactic errors.

It is convenient now to have two observers, O_1 and O_2, one to watch the tonometer, and the other to manipulate the beakers of warm and of ice-cold water, and the wire terminals of the storage cell, and to observe the bubble. Let us suppose that this is lying in the tube with its upper end at the platinum wire. On setting the warm water Y in place, the bubble closes in, and, just before it disappears, will begin to float up the tube, disappearing, however, before it has risen more than 1 or 2 centims. At the moment of its disappearance, *and not before*, does the observer of the tonometer notice a sudden rise of the mercury; this shows the freedom from residual undissolved gas. The warm liquid is quickly removed, and the ice-cold liquid substituted; this causes the mercury in the tonometer to fall, and its observer O_1 calls aloud the divisions as it passes them; when the tension is approached at which it is desired to observe the extension, the ice-cold water is removed, so that the cooling proceeds more slowly, and while O_1 is carefully watching and calling the readings, O_2 makes a momentary completion of the electric circuit, which results in the sudden reappearance of a bubble of definite length, which he reads off while observer O_1 notes the height to which the mercury in the tonometer has again risen.

It is not always possible in practice to prevent the liquid from letting go its hold too soon, and it often happens that the bubble forms in one of the bulbs, and not in the tube, where only its volume can be read off, but this fact need not prevent an observation of the volume from being made, for, if the observer O_2 only heats the platinum wire for an instant, the liquid below it, which has been clinging to it, separates, owing to its weight, and at once a bubble of hot vapour is formed, which, by its greater pressure, causes the bubble in the bulb, which also is at a lower level, to close up and vanish. Thus the bubble can be instantly transferred to the place where it can be measured.

The method of liberating the bubble where it was wanted by means of the heated platinum wire was only hit upon after many other methods had been tried unsuccessfully, but I had not foreseen that it would carry with it the advantage of enabling the observer to transfer the bubble in the manner just mentioned. This was a piece of good luck.

Corrections to be applied to the Measure of the Bubble.

(1.) *Correction for Capillary Curvature.*—This was applied in the usual way, the ends being considered as hemispheres of the diameter of the tube, which was 2·20 millims.

(2.) *Correction for Inequality of the Bore near the Wire.*—This was determined by floating the bubble when very small to a point in the tube 3 or 4 centims. below the platinum wire, and then allowing the whole liquid to cool to a steady temperature, thus forming a long bubble in a uniform part of the tube. The length of this bubble was then carefully measured, and then this bubble was "transferred" in the manner mentioned, so that its upper end was at the wire. The difference of the lengths gives the correction to be applied. The manner in which the upper end of the bubble was entangled with the platinum wire was not, however, always the same, but appeared to be always one or other of two configurations, and the correction to be applied for the variation of bore was + 1·55 millims., or + ·015 millims., according to the configuration which was observed and recorded on each occasion.

Sources of Error in Determining the Length of the Bubble.

(1.) It was often impossible so to regulate the influx or efflux of heat as to secure that a stationary stage had been reached when the bubble was formed. Thus the liquid generally continued to shrink by cooling after having let go the walls, and on this account the bubble required to be read immediately.

(2.) On the other hand, if the reading was made too soon there was not time for the liquid still clinging to the side of the tube to drain down.

Each of these two sources of error tends to make the bubble appear too long, *i.e.*, the measure of the strain too great; and it is, without doubt, to their action that the irregularity of the observations must be chiefly attributed. Fortunately, the degree of steadiness of the tonometer just before the liquid is made to let go enables the observer to discriminate somewhat between "good" and "less certain" observations.

Difficulties and General Observations.

It is in this connection, and before passing to the measurements themselves, that the difficulties attendant on the experiments may be alluded to. In the first place, considerable tensions (above 12 atmospheres) are not easy to attain, the liquid letting go its hold unexpectedly and too soon; sometimes this will continue for an hour or two, the bubble appearing perpetually in the same place in one or other of the bulbs, or at one of the bends. This is, perhaps, attributable to the liberation of air from minute crevices in the glass, which air has to be dispersed before adhesion can be re-established. For this purpose I have often found the application of ice or of a freezing mixture to the affected part to be efficacious, especially when accompanied by judicious sharp tapping of the apparatus, so as to secure an impulsive pressure of the liquid against the surface at the non-adherent part of the glass, whereby I imagine that the minute bubble or film of air is the better dispersed. It is also useful, by manipulation and variation of the temperature of different parts of the liquid, to produce currents which sweep away from the narrow tubes into the wider bulbs portions of the liquid that may have become somewhat charged with dissolved gas. Yet at all times the behaviour of the liquid, as regards its adhesion, is somewhat uncertain. Thus, there may be no difficulty in reaching, time after time, a tension, say, of 9 atmospheres, the liquid always separating from the wall at some particular place; and when, after an hour or more of patient manipulation, the observer is beginning to think the disease incurable, and that the apparatus must be reopened and the whole process of filling and boiling-out gone through again, the difficulty will unexpectedly disappear, or change.

Meanwhile, and especially when high tensions have been reached, there is constant danger that the suddenness of the release may break the whole apparatus; and this is the more to be feared when the release is an accidental one, taking place in one of the bulbs, than when it is intentionally effected at the wire in the narrow tube, for in the latter case the friction of the long column of liquid against the walls of the tube appears to act as a brake, and the shock to the apparatus is less sudden. Four pieces of apparatus similar to that described broke in this way, under the sudden release, before I was able to obtain any determinate measures. I have endeavoured to diminish this danger by connecting the two stems, AG and DE (fig. 1), and the cross-connecting tube as rigidly as possible to a light board when experimenting, so as to prevent these parts acquiring any considerable momentum.

It must not, however, be supposed that the condition of the liquid, when in a state of tension, is necessarily nearly unstable. Thus, before the method of detachment by heating the platinum wire was hit upon, it was often very difficult to effect a release. With the tonometer indicating a pull of from 8 to 12 atmospheres, the liquid could not be made to let go its hold by moving the apparatus about, nor by means of taps and jars as violent as it seemed safe to give, nor by strong local heating of the narrow tube. Meanwhile, specks of suspended impurity could be seen floating about in the interior, proving the liquidity of the substance; and, when the release did take place, the immediate rise of the mercury in the tonometer to its normal position showed that no mistake had been made in the measurement of the tension to which the liquid was all the time subjected.

The Observations.

The accompanying diagram (Plate 10) is a graphic record of all the observations that I succeeded in obtaining.

The first set were taken on May 17, 1890, and are distinguished thus ⊕. It was in making these that I became fully aware of the importance of aiming at steadiness of temperature before liberating the bubble, so as to permit of a little waiting till the liquid on the sides of the tube had drained down. Hence these measures are, with apparently one exception, probably all a little too high. The observations recorded + were made on May 19, with all possible care; two of them low down in the diagram were recorded as doubtful, and are marked (?). Those marked ⊕ correspond to cases in which the bubble formed at the top of the tube FE (fig. 1), from which position it could not be "transferred" to the wire. This upper portion of the tube had not been calibrated with special attention, since its employment had not been anticipated, and the subsequent breakage of the apparatus prevented this being done afterwards. Nevertheless, I have thought it best to introduce the observations into the diagram. An eye estimate had to be made of the contraction of the tube due to the sealing at E, but in long bubbles an error on this score must have been comparatively small. It will be observed, however, that all the observations in question fall rather below the mean curve. The observations marked ⊞ were made on May 22, in order to fill in regions about which information was still needed. All of these were recorded as "good" observations.

I interpret these observations as representing the straight line through the origin drawn in the diagram and making an angle with the axis of stress, whose tangent is 0·700.

The vibrations set up, and the shock sustained by the apparatus on the sudden release from the highest tension reached (over 17 atmospheres) were so violent that it seemed unsafe to proceed further before making such observations as were necessary to eliminate the effect of the yielding of the glass itself, and which could not be made if the vessel were broken.*

The vessel was therefore opened, and the alcohol in it subjected to a pressure of 12·38 atmospheres in excess of the external atmospheric pressure, corresponding to 40 tonometer divisions, and the *apparent* compression of the liquid was observed. The measures of the retreat of the end of the column along the tube were

	millims.
	28·19
	27·53
	28·36
	28·02
	27·05
	27·6
(Specially trustworthy)	27·78
Mean .	27·79

The observed apparent extension *in the same vessel* under an equal tension, as given by the line of the diagram, is 27·8 millims. The closeness of the agreement is, of course, in part fortuitous, for the line of the diagram cannot be placed with extreme precision; but the practical coincidence of the two numbers is a very satisfactory confirmation of the view that the observations correspond to a straight line, and, since the small yielding of the nearly rigid glass vessel must be proportional to the stress, permits us to draw the conclusion that *in the neighbourhood of the zero pressure the absolute coefficient of volume elasticity of alcohol is the same for extension as for compression, and so far as the observations show is constant between a pressure of* $+ 12$ *and* $- 17$ *atmospheres.*

The best way to obtain the *absolute* value of the coefficient is probably by direct experiments on the compressibility of alcohol in the neighbourhood of the zero point. The value given in LUPTON'S tables is $\frac{1}{11\cdot 4}$ per 10^6 grms. per sq. centim., or ·0000906 per atmosphere of 1033·3 grms. Mr. SKINNER by recent experiments made in the Cavendish Laboratory on alcohol at 13°·5 C. between 1 atmosphere and 1·3 atmospheres obtains the value ·000093 per atmosphere.

It seemed desirable, if only to serve as a check on the experiments, to mention the attempt that I made to determine directly the yielding of the glass vessel. For this purpose the alcohol was boiled out until only a very small residue was left, when mercury was allowed to enter which completely filled up the whole instrument. This mercury, whose compressibility is some fifty times less than that of alcohol, was then

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* As a matter of fact, the stem DE was unfortunately broken in setting up the apparatus for exhibition in the rooms of the Royal Society, at the Soirée of June 18, 1890, but this was afterwards repaired without any interference with the neighbouring bulb.

subjected to pressures of 10 and 12 atmospheres (above the external atmospheric pressure), and the retreat of the end of the mercury column along the tube was noted. The following measures of this recession were obtained for a pressure of 10 gauge-atmospheres of 15 pounds per sq. inch.

	millims.
Direct observation at 10 atmospheres . . .	7·31
	7·33
From observation at 12 atmospheres . . .	7·40
	7·40
	7·1
Mean	7·31

The mean value is equivalent to 7·22 millims. per 10 atmospheres of 1033 grms. per sq. centim.

The measures were somewhat unsatisfactory on account of (1) the smallness of the total length to be measured, (2) the deformation of the meniscus due partly to a certain want of cleanness at the surface which dragged along the glass, (3) a tendency to entangle bubbles of alcohol between the mercury and the sides of the tube owing to the churning as the end of the column moved to and fro. This ultimately caused a separation of the column into segments, and must have had the effect of diminishing the apparent recession. (4) It should also be mentioned that the end of the column was situate in the upper part of the tube FE, which, owing to the subsequent breakage of the apparatus, I was not able to specially calibrate, but which, if we may judge from the measures of the bubbles, was probably somewhat wider than the part below the wire. On this account also the measured result is probably rather too small.

If the mercury were quite incompressible, and if there were no residual alcohol in the vessel, this number would represent, in millims. of the tube, the yielding of the glass per 10 atmospheres, but by reason of the corrections required on these two accounts this length is reduced to 6·626 millims.*

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* The calculation of these corrections was made as follows :—Let the change required in the volume of the glass per 10 atmospheres, be v_g millims. of the tube, and let the corresponding changes in the volume of the mercury, and of the residual alcohol be v_m and v_a millims. of the tube respectively, and let the observed apparent alteration of volume be (a) millims. of tube. Then

$$v_g + v_m + v_a = a \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (\text{i.}).$$

Let the vessel, when full, contain n times as much alcohol as this residue. Then admitting extension and compression under numerically equal stresses to have been proved equal (p. 367), we have

$$v_g + nv_a = \text{A} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (\text{ii.}),$$

Subtracting this amount from the ordinate of the diagram corresponding to 10 atmospheres, the remainder, of length 16·17 divisions, gives the corrected value of the absolute strain, and is equivalent to 10·67 ten-thousandths of the whole volume, per stress of 10 atmospheres, or to ·0001067 per atmosphere, a result which is about 15 per cent. higher than that of Mr. SKINNER. From the nature of my observations on this point, I do not think that much importance is to be attached to this discrepancy, which represents a possible error of 2 millims. to be divided between the measure of the length of the bubble on the release from 10 atmospheres tension, and the measure of the recession of the mercury in the experiments just quoted.

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where A is the observed apparent alteration of volume in millims. of tube, due to 10 atmospheres tension, when the vessel was full.

Hence

$$v_a = \frac{A}{n} - \frac{v_g}{n}.$$

Substituting this value for v_a in (i.), and multiplying up

$$(n-1)\, v_g + n v_m = na - A,$$

or

$$v_g = \frac{n}{n-1}\, a - \frac{n}{n-1}\, v_m - \frac{A}{n-1},$$

in which expression $\frac{n}{n-1} v_m$ is a small corrective term due to the compressibility of the mercury, while $\frac{A}{n-1}$ is a small corrective term due to the compressibility of the residual alcohol present with the mercury in the bulb.

The numerical values required were obtained as follows:—

	grms.
The weight of mercury and alcohol together was found to be . . .	769·3
,, ,, alone	768·38
Therefore, weight of residual alcohol alone =	0·92

Hence the volume of the the mercury alone = 56·5 cub. centims., and the volume of the residual alcohol alone = 1·15 cub. centims.

The volume of the alcohol used in the stretching experiments was 58·5 cub. centims., therefore

$$n = \frac{58·5}{1·15} = 50·6, \text{ and } n - 1 = 49·6.$$

The observed recession of the mercury (a) = 7·22 millims. of tube, as already stated in the text.

A = 22·7 millims. of tube, as read off from the line in the diagram.

v_m = ·273 millims. of tube (deduced from the value of the compressibility of mercury given in LUPTON's tables, viz., $\frac{1}{552·5}$ per 10^6 grms. per sq. centim., and the fact that $\frac{1}{10000}$ of the volume of mercury employed = $\frac{56·5}{58·5}$ of 1·513 millim. of the tube).

Hence

$$v_g = \frac{50·6}{49·6} \times 7·22 - \frac{50·6}{49·6} \times ·273 - \frac{22·7}{49·6}.$$

$$= 7·36 - 0·278 - 0·456.$$

$$= 6·626 \text{ millims. of tube.}$$

General Remarks on the Experiments.

It will be noticed that the manner of experimentation has the disadvantage that it does not permit of the whole of the liquid under tension being at the same temperature. The mean temperature was probably at no time more than 5° removed from 16° C. I am now endeavouring to make an apparatus in which it shall be possible to maintain the portion of the liquid, whose stretching is to be measured, at any desired constant temperature, whereby also the chief difficulty in the way of more exact measurements will be overcome.

As regards the conclusion reached, that the changes of volume of a mass of liquid are equal for numerically equal increments of pressure, whether positive or negative, it may be justly observed that this was only to be expected. No one, however, could predict that the coefficient of extensibility would remain practically constant up to tensions of 17 atmospheres, and nothing but further experiment can decide what changes may take place in its value as the tension is increased.

Bearing of the Observations on the Theory of Surface-Tension.

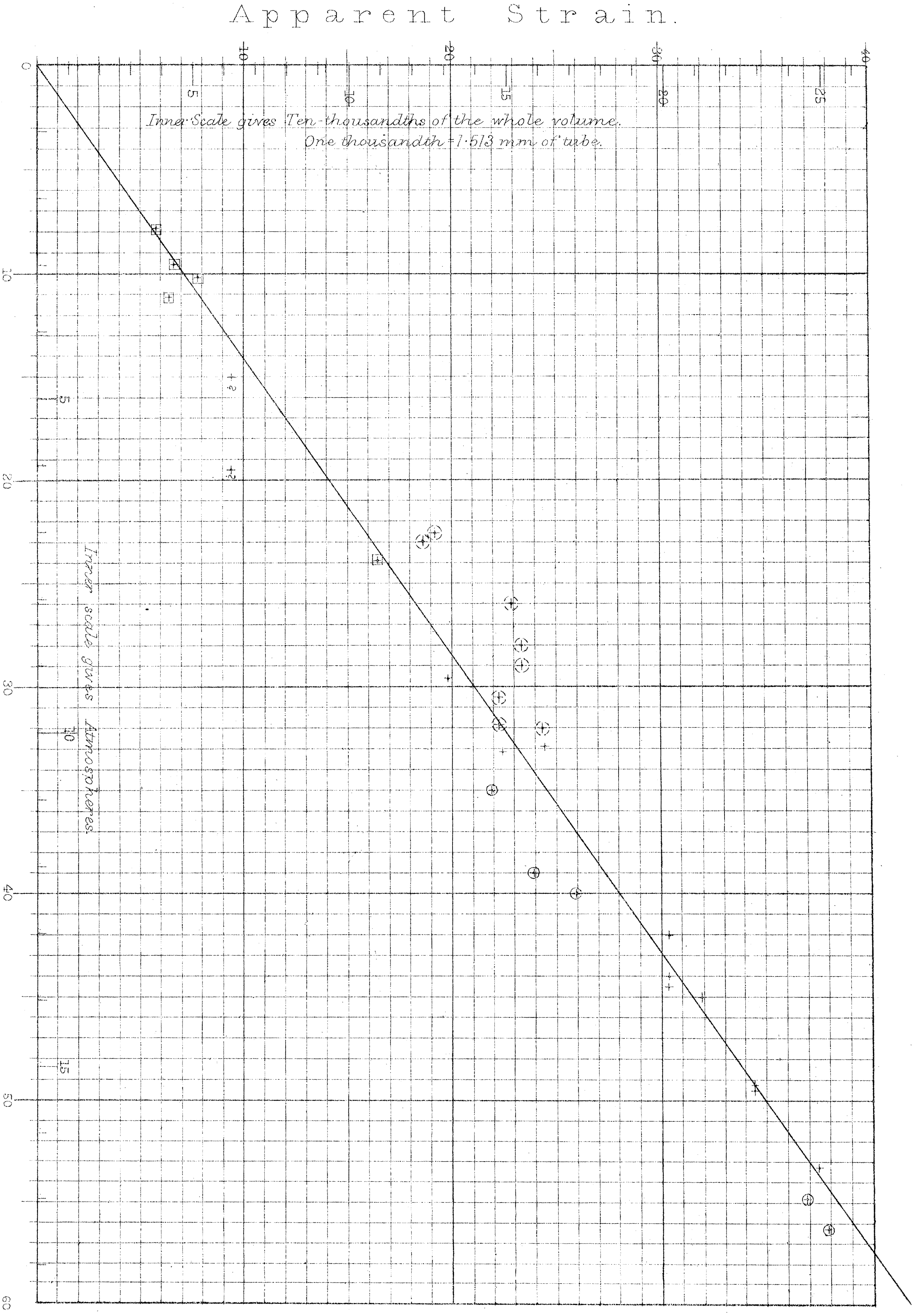
The unequivocal proof that a liquid can exist in stable equilibrum, in a state of isotropic tensile strain, has a bearing on the theory of surface forces in fluids. For it can be shown to be necessary for equilibrium that a compressible liquid shall be, close to the free surface, less dense than in the interior; in other words, the surface layers are in a condition to which interior liquid could be brought by stretching it, and are, therefore, a seat of energy in precisely the same way that stretched liquid is a seat of energy. A theory, such as that of LAPLACE, which assumes uniform density precludes us from admitting in the material itself any such modification correspondent to the surface energy, and drives us to seek it in the condition of the superjacent ether film.

Note on a Curious Phenomenon of Adhesion between two Solids immersed in a Stretched Liquid.

Desiring to ascertain whether an air-free liquid would adhere under tension, as well to a metal as to glass, I enclosed a small piece of folded sheet copper in a glass bulb, which was then filled with boiled-out, air-free alcohol. Experiments with this showed strong adhesion to the copper, as well as to the glass, provided the vessel was kept still, but any agitation at once caused the stretched liquid to let go its hold at the place of contact of the copper and the glass. Close attention showed that the copper seemed to "grow to the glass" at the points of contact, when the surrounding liquid was in a state of tension. This led to experiments on bulbs, with smaller bulbs of glass inside, and in all cases the same phenomenon was observed: when the liquid was stretched, the loose bulb attached itself to the side of the vessel. The equilibrium was, however, very unstable. The release of the liquid took place on the slightest jar, the bubble always appearing at the contact of the solid with the wall, and the loose piece being generally tossed up when the rupture took place. I succeeded best with one small irregular bulb with a projecting stem; this could be gently waved about in the stretched liquid, while the foot of the stem adhered to a point on the side of the containing vessel (showing incidentally that considerable currents may exist in a stretched liquid). The explanation of this phenomenon, which at first puzzled me, may, I think, be given as follows.

At the surface of glass, any liquid, such as alcohol, which wets it, is condensed. Over the area of contact of the loose piece with the side of the vessel, this condensed film is probably somewhat thinner than elsewhere, being squeezed out (as by hydrostatic pressure) by the cohesive attraction between the two solids. When the liquid is in a state of tension, there is everywhere a demand for liquid to stretch, which is met by any approach of the cohering surfaces, for such approach will increase both the closeness and the area of contact, and yield a supply of hitherto condensed liquid, by diminution of the surface over which it has been condensed. Any displacement in the direction of further approach will therefore be resisted only by the elasticity of the solids, called into play by the deformation at the area of contact. Under ordinary circumstances such displacement is resisted also by the hydrostatic pressures. To understand the instability, it is only necessary to remember that, with a comparatively rigid substance like glass, small relative motion of the parts may generate very large impulsive stresses at points where the relative motion is prevented.

1 division = 1 mm. of stem of fine tube = ·000066095 of whole volume.



Stretching of ETHYL ALCOHOL.

West, Newman sc.